By use of the integrating factor $\exp\int(n/X)dX$, Eq. (8) is integrated to

$$\Delta = [\Lambda X/(n+1)] + C \tag{9}$$

Since $C = 0$ at $X = 0$, Eq. (9) can be expressed as

$$\frac{1}{1+n}\cdot\frac{\rho_\omega u_\infty X}{\mu_\omega} = \frac{D}{K^3(1+B-A^2)^{1/2}} \times \left\{a^2 \exp\left[\frac{a}{A}(\sin^{-1}\alpha + \sin^{-1}\beta)\right]\right\} \tag{10}$$

Comparison of Eq. (10) with Eq. (4) shows that the axially symmetric heat transfer is the same as the two-dimensional value when the Reynolds number is divided by $(1 + n)$. Note that for $n = 1$ the preceding reduces to Eq. (6), which agrees exactly with Eq. (23) of Ref. 2. Finally, since the turbulent shear stress varies inversely with the $\frac{1}{5}$ power of the Reynolds number, then

$$(\tau_A/\tau_{2D}) = (1+n)^{1/5} \tag{11}$$

The assumptions that $\delta \ll r$ and $\partial P/\partial X = 0$ are valid only at $X = \infty$. However, Ref. 1 shows that for an expression similar to Eq. (11) (which was also developed at $X = \infty$) the results are acceptable at smaller values of X. This fact tends to indicate (although no proof is available) that the turbulent results shown here will give similar accuracy at the more practical values of X.

Thus, a simple relationship has been developed to transform two-dimensional shear stress to axially symmetric shear stress for bodies of the form $r = cx^n$.

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Backside Temperatures of an Internal Insulator in a Solid-Propellant Motor

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Nomenclature

R = average erosion rate
t_i = initial temperature of insulator
t_d = decomposition temperature of insulator
t_b = backside temperature of insulator
x_i = initial insulator thickness
u = insulator thickness at time θ
θ = exposure time of insulator surface
α = thermal diffusivity of insulator

SOLID-PROPELLANT rocket motors are internally insulated to minimize strength degradation of the pressurized case during propellant burning time. The solid propellant is an effective self-insulator and helps to protect the motor case from the chamber environment until the flame front reaches the internal insulator surface.[1] The exposure time of the internal insulator surface to the hot and erosive chamber gases can be predicted from a knowledge of the propellant design and burning rate. A simple end-burning propellant design allows the insulator surface to become gradually exposed, whereas internal-burning designs, such as a wagon wheel or star, result in almost the entire insulator surface being exposed simultaneously at the end of the propellant burning time. Usually, a more complex propellant design is required, and the exposure time of the insulator surface varies circumferentially around the motor case and longitudinally along the motor case. The internal insulator will undergo erosion, which can be measured in a post-firing inspection. An average erosion rate for any point on the insulator may be calculated from the measured erosion and exposure time, or the erosion rate may be estimated from prior data. This technical note reports an investigation in which the average erosion rate of a silica-filled Buna-N internal insulator and exposure time of the insulator surface to the chamber environment were related to estimate the backside temperature of the insulator. A comparison was made between predicted and experimental insulator backside temperatures for various initial thicknesses in a full-scale, modified, double-base, solid-propellant rocket motor. The experimental temperature data were obtained by using no. 28 American wire gage iron-constantan thermocouples.

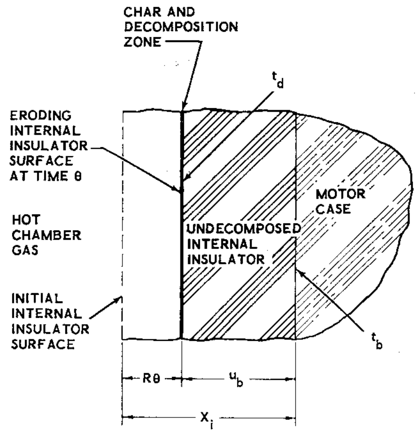

Fig. 1 Diagram of eroding insulator at time θ.

Literature contains many articles on heat conduction with a moving boundary or heat source. Eckert,[2] Schneider,[3] and Carslaw and Jaeger[4] treat various cases. This investigation is based on the assumption that the internal insulator erodes at a constant average rate R at a particular point on the insulator during exposure to hot chamber gases. A large temperature gradient exists through the surface film, insulator char, and decomposition zones. Thin char and decomposition zones are assumed to have a constant thickness. Under these conditions, temperature of the cool side of the decomposition zone, t_d, is independent of chamber environment, temperature, pressure, velocity, etc., which affect the rate of erosion. The insulator backside temperature is t_b.

A moving coordinate system is used with the origin at the exposed surface of the insulator. With this system, the backside surface is moving at the constant rate R toward the insulator surface. A quasi-steady state exists with $\partial t/\partial\theta = 0$ at any point in the material referred to the moving coordinate system. This system is illustrated in Fig. 1. It is assumed that thermal properties of the undecomposed insulator are constant and that heat flow is unidirectional and follows the unsteady-state conduction law

$$\partial t/\partial\theta = \alpha(\partial^2 t/\partial x^2) \tag{1}$$

With a boundary moving at a constant rate, u is related to x and θ by

$$u = x - R\theta \tag{2}$$

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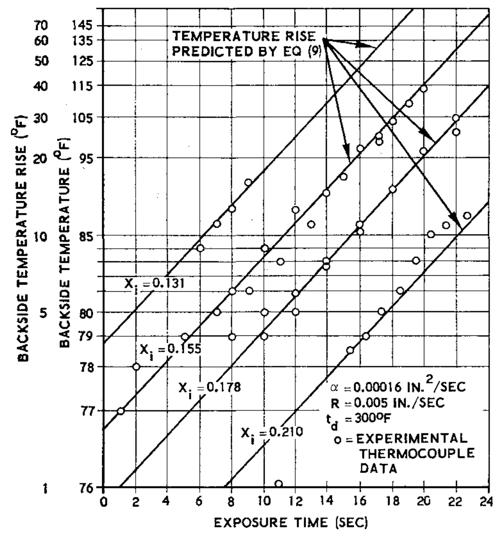

Fig. 2 Comparison of predicted and experimental backside temperatures for various thickness insulators.

Therefore,

$$\partial u/\partial \theta = -R \tag{3}$$

and noting that

$$\partial^2 t/\partial x^2 = \partial^2 t/\partial u^2 \tag{4}$$

and

$$\frac{\partial t}{\partial \theta} = \frac{\partial u}{\partial \theta}\frac{\partial t}{\partial u} + \frac{\partial t}{\partial \theta'}\frac{\partial \theta'}{\partial \theta} \tag{5}$$

with $\partial \theta'/\partial \theta = 1$, substituting Eq. (3) into Eq. (5) gives

$$\frac{\partial t}{\partial \theta} = -R\frac{\partial t}{\partial u} + \frac{\partial t}{\partial \theta'} \tag{6}$$

substituting Eqs. (6) and (4) into Eq. (1) gives

$$\frac{d^2 t}{du^2} + \frac{R}{\alpha}\frac{dt}{du} = 0 \tag{7}$$

by noting that $\partial t/\partial \theta' = 0$ in the quasi-steady state. Integration of Eq. (7) and use of semi-infinite boundary conditions, $t \rightarrow t_i$ as $u \rightarrow \infty$ and $t = t_d$ when $u = 0$, yields the following for the backside temperature rise:

$$t_b - t_i = (t_d - t_i)\exp(-Ru_b/\alpha) \tag{8}$$

Employing Eq. (2), which relates u_b, x_i, and θ, yields the following:

$$t_b - t_i = (t_d - t_i)\exp(-Rx_i/\alpha + R^2\theta/\alpha) \tag{9}$$

By use of Eq. (9), it is possible to estimate the backside temperature rise of an insulator as a function of exposure time for various initial thicknesses in a solid-propellant rocket motor.

The value of t_d, used in Eq. (9), may be determined from experimental data. In this investigation, data showed it to be approximately 300°F. Physically, this value approximates the start of thermal decomposition in silica-filled Buna-N rubber.

In Fig. 2, experimental backside temperature for various exposure times and initial thicknesses are compared with those predicted by Eq. (9). The predicted backside temperatures and the experimental data are in close agreement, within 3°F, at any reported exposure time.

Agreement of the predicted and experimental data indicates that a useful approximate relationship has been developed to estimate rapidly the backside temperature of silica-filled, Buna-N internal insulators in solid-propellant rocket motors. This relationship can be used in design applications for rapid selection of optimum insulator and motor-case thicknesses.

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Flutter of Multibay Panels at Supersonic Speeds

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In Ref. 1, Dowell has carried out a comprehensive study of the flutter of two-dimensional multibay panels at high supersonic Mach numbers. The stability boundaries presented in Ref. 1 for the two-bay and four-bay configurations differ substantially from earlier results obtained by the present author. Dowell attributes the differences to a lack of convergence related to approximations in the previous collocation (or lumped-parameter) solutions. This was not the case, and it is the purpose of this note to review the difficulties in the previous solutions and to correlate the correctness of Dowell's results by proper application of the collocation method.

The two-bay panel was originally investigated in the author's thesis,[2] and the results have been reported in Refs. 3 and 4; the subsequent investigation of the four-bay panel was reported by Fung[4] in his survey of theories and experiments on panel flutter. The method of analysis was a numerical collocation solution of the aeroelastic integral equation expressed in terms of a structural influence coefficient matrix (SIC), a mass matrix, and an aerodynamic influence coefficient matrix (AIC), which permitted construction of the conventional required damping vs velocity flutter stability curve. The particular collocation formulation was developed in Ref. 5, and the panel AIC's have been reviewed in Ref. 6. The two-bay and four-bay panels are considered in turn in the following, since each encountered a different difficulty in its solution.

In reviewing the two-bay panel calculations, the question of convergence was considered first. In Ref. 2 each bay was divided into ten segments and nine collocation control points, the SIC's and AIC's (including *all* unsteady terms in the panel pressure expression) were generated by desk calculations (the mass matrix was a unit diagonal), and the North American Aviation, Inc., high-speed digital computer program based on Ref. 5 was used for the flutter solution. The present state of automation includes not only a faster and more accurate flutter solution[7] but also programs for the generation, in punched-card format, of SIC's[8] and panel AIC's.[9] It was therefore feasible to attempt verification of the sufficiency of the original 18 degree-of-freedom analysis of the two-bay panel by doubling the number of divisions of each bay (20 segments and 19 control points). The 38 degree-of-freedom analysis demonstrated the adequacy of the original division. It then became necessary to look elsewhere for the source of the discrepancy. It was soon dis-

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